

## Übungsblatt 10 Gewöhnliche Differentialgleichungen

### Gewöhnliche Differentialgleichungen, Grundbegriffe:

#### A1.

Prüfen Sie zu der gegebenen DGL, ob die gegebene Funktion die Lösung der DGL ist:

$$x = \ln y' + \sin y', \quad x = \ln t + \sin t, \quad y = t(1 + \sin t) + \cos t$$

### Gewöhnliche Differentialgleichungen 1. Ordnung

#### Integration durch Trennung der Variablen

#### A2

1.  $2(x-1)dx + 3y^2 dy = 0, \quad y(2) = 1$

2.  $y' = x(y-1), \quad y(0) = \frac{1}{2}$

3.  $y' + x^2(y-2) = 0, \quad y(0) = -2$

#### A3

1.  $y' - x^2 = 0, \quad a) y(-3) = 1, \quad b) y(2) = 5$

2.  $y' = x^2 + 4x - 3, \quad a) y(1) = -2, \quad b) y(-3) = 3$

3.  $y' = 4 + y, \quad a) y(0) = 1, \quad b) y(1) = 3$



## Integration durch Substitution

**A4:**

1.  $y' = 4x - y$ ,      a)  $y(2) = 4$ ,      b)  $y(0) = -2$

2.  $y' = x + y$ ,      a)  $y(0) = 2$ ,      b)  $y(-2) = 3$

## Variation der Konstanten

**A5**

1.  $y' + y = m e^{-nx}$ ,     $y(0) = 1$ ,

a)  $m = n = 1$ ,    b)  $m = 1, n = 2$ ,    c)  $m = 2, n = 1$

2.  $y' + \frac{y}{x} = \sin(mx)$ ,     $y(\pi) = 1$ ,    a)  $m = 1$ ,    b)  $m = 2$

3.  $xy' + my = x$ ,     $y(1) = -1$ ,    a)  $m = 1$ ,    b)  $m = 2$ ,    c)  $m = 4$

4.  $xy' + my = x^2$ ,     $y(1) = 0$ ,    a)  $m = 2$ ,    b)  $m = -2$

5.  $xy' - y = x^m$ ,     $y(1) = 1$ ,    a)  $m = 1$ ,    b)  $m = 2$ ,    c)  $m = 3$



Lösungen – Übungsblatt 10:

1. **Gewöhnliche Differentialgleichungen, Grundbegriffe:**

$$x = \ln y' + \sin y', \quad x = \ln t + \sin t, \quad y = t(1 + \sin t) + \cos t$$

$$y' = y'_x = \frac{y'(t)}{x'(t)}$$

$$x'(t) = \frac{1}{t} + \cos t, \quad y'(t) = 1 + \sin t - \sin t + t(1 + \cos t) = 1 + t \cos t$$

$$y' = \frac{1 + t \cos t}{\frac{1}{t} + \cos t} = \frac{1 + t \cos t}{\frac{1}{t}(1 + t \cos t)} = t, \quad t > 0$$

$$\Rightarrow \ln t + \sin t = \ln t + \sin t$$

Lsg.

2.

**Gewöhnliche Differentialgleichungen 1. Ordnung**

**Integration durch Trennung der Variablen**

1.  $2(x-1)dx + 3y^2 dy = 0, \quad y(2) = 1$

$$2 \int (x-1) dx + 3 \int y^2 dy = 0, \quad (x-1)^2 + y^3 = C$$

Allgemeine Lösung:  $(x-1)^2 + y^3 = C$

$$y(2) = 1 : C = 2, \quad (x-1)^2 + y^3 = 2$$

2.  $y' = x(y-1), \quad y(0) = \frac{1}{2}$

$$\int \frac{dy}{y-1} = \int x dx, \quad \ln |y-1| = \frac{x^2}{2} + \ln |C|, \quad \ln \left| \frac{y-1}{C} \right| = \frac{x^2}{2}$$

Allgemeine Lösung:  $y = e^{\frac{x^2}{2}} + C$

$$y(0) = \frac{1}{2} : C = -\frac{1}{2}, \quad y = e^{\frac{x^2}{2}} - \frac{1}{2}$$

Noch zu Lsg.2:

$$3. \quad y' + x^2(y - 2) = 0, \quad y(0) = -2$$

$$\int \frac{dy}{y-2} = - \int x^2 dx, \quad \ln |y-2| = -\frac{x^3}{3} + \ln |C|$$

$$\text{Allgemeine Lösung: } y = 2 + C e^{-\frac{x^3}{3}}$$

$$y(0) = -2 : C = -4, \quad y = 2 - 4 e^{-\frac{x^3}{3}}$$

Lsg.

3.

$$1. \quad y' - x^2 = 0, \quad a) y(-3) = 1, \quad b) y(2) = 5$$

$$\text{Allgemeine Lösung: } y = \frac{x^3}{3} + C$$

$$a) y(-3) = 1 : y = \frac{x^3}{3} + 10, \quad b) y(2) = 5 : y = \frac{x^3}{3} + \frac{7}{3}$$

$$2. \quad y' = x^2 + 4x - 3, \quad a) y(1) = -2, \quad b) y(-3) = 3$$

$$\text{Allgemeine Lösung: } y = \frac{x^3}{3} + 2x^2 - 3x + C$$

$$a) y(1) = -2 : y = \frac{x^3}{3} + 2x^2 - 3x - \frac{4}{3}$$

$$b) y(-3) = 3 : y = \frac{x^3}{3} + 2x^2 - 3x - 15$$

$$3. \quad y' = 4 + y, \quad a) y(0) = 1, \quad b) y(1) = 3$$

$$\text{Allgemeine Lösung: } y = C e^x - 4$$

$$a) y(0) = 1 : y = 5 e^x - 4, \quad b) y(1) = 3 : y = \frac{7}{e} e^x - 4 = 7 e^{x-1} - 4$$



Lsg.

## Integration durch Substitution

4.

1.  $y' = 4x - y$ ,    a)  $y(2) = 4$ ,    b)  $y(0) = -2$

$$u = 4x - y, \quad \frac{du}{dx} = 4 - \frac{dy}{dx} \Leftrightarrow u' = 4 - y'$$

$y' = 4x - y = u$  in Eq.  $u' = 4 - y'$  einsetzen:  $u' = 4 - u$

$$\frac{du}{dx} = 4 - u, \quad \int \frac{du}{4-u} = \int dx, \quad \ln(4-u) = x + \ln C, \quad 4-u = C e^x$$

weiter folgt die Rücksubstitution

Allgemeine Lösung:  $y = C e^{-x} + 4x - 4$

a)  $y(2) = 4$  :  $y = 4x - 4$ ,    b)  $y(0) = -2$  :  $y = 2 e^{-x} + 4x - 4$

2.  $y' = x + y$ ,    a)  $y(0) = 2$ ,    b)  $y(-2) = 3$

Allgemeine Lösung:  $y = C e^x - x - 1$

a)  $y(0) = 2$  :  $y = 3 e^x - x - 1$ ,    b)  $y(-2) = 3$  :  $y = 2 e^{x+2} - x - 1$

Lsg.

## Variation der Konstanten

5.

1. a)  $m = n = 1$  :  $y' + y = e^{-x}$ ,     $y = (x + C) e^{-x}$

$y(0) = 1$  :  $y = (1 + x) e^{-x}$

b)  $m = 1, n = 2$  :  $y' + y = e^{-2x}$ ,     $y = (-e^{-x} + C) e^{-x}$

$y(0) = 1$  :  $y = (2 - e^{-x}) e^{-x}$

c)  $m = 2, n = 1$  :  $y' + y = 2 e^{-x}$ ,     $y = (2x + C) e^{-x}$

$y(0) = 1$  :  $y = (1 + 2x) e^{-x}$

2. a)  $m = 1 : y' + \frac{y}{x} = \sin x, \quad y = \frac{1}{x} (\sin x - x \cos x + C)$   
 $y(\pi) = 1 : y = \frac{1}{x} (\sin x - x \cos x)$
- b)  $m = 2 : y' + \frac{y}{x} = \sin(2x), \quad y = \frac{1}{4x} (\sin(2x) - 2x \cos(2x) + 4C)$   
 $y(\pi) = 1 : y = \frac{1}{4x} \sin(2x) - \frac{1}{2} \cos(2x) + \frac{3\pi}{2x}$
3. a)  $m = 1 : xy' + y = x, \quad y = \frac{x}{2} - \frac{C}{x}, \quad y(1) = -1 : y = \frac{x}{2} - \frac{3}{2x}$
- b)  $m = 2 : xy' + 2y = x, \quad y = \frac{x}{3} + \frac{C}{x^2}, \quad y(1) = -1 : y = \frac{x}{3} - \frac{4}{3x^2}$
- c)  $m = 4 : xy' + 4y = x, \quad y = \frac{x}{5} + \frac{C}{x^4}, \quad y(1) = -1 : y = \frac{x}{5} - \frac{6}{5x^4}$
4. a)  $m = 2 : xy' + 2y = x^2, \quad y = \frac{x^2}{4} + \frac{C}{x^2}, \quad y(1) = 0 : y = \frac{x^2}{4} - \frac{1}{4x^2}$
- b)  $m = -2 : xy' - 2y = x^2, \quad y = (\ln x + C)x^2, \quad y(1) = 0 : y = x^2 \ln x$
5. a)  $m = 1 : xy' - y = x, \quad y = (\ln x + C)x, \quad y(1) = 1 : y = (\ln x + 1)x$
- b)  $m = 2 : xy' - y = x^2, \quad y = (x + C)x, \quad y(1) = 1 : y = x^2$
- c)  $m = 3 : xy' - y = x^3, \quad y = \frac{x^3}{2} + Cx, \quad y(1) = 1 : y = \frac{x^3}{2} + \frac{x}{2}$