Computer-aided study of imperfect pulse vaccination model

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Mathematical epidemiology is concerned with the mathematical models of the spread of diseases in populations. Our aim is to control the disease with different tools, such as quarantine, medical care or vaccination program. In this talk, we investigate an S-I-V model with imperfect vaccine, i.e., vaccinated individuals can also be infected. We consider pulse vaccination (proved to be more effective than the continuous one), that means we vaccinate fraction φ of population at times $t = nT$. The model reads as

$$\begin{cases} S'(t) = \mu - \beta S(t) I(t) - \mu S(t) + \gamma I(t) + \theta V(t) \\ I'(t) = \beta S(t) I(t) - (\mu + \gamma) I(t) + \sigma \beta V(t) I(t) & \text{if } t \neq nT, \\ V'(t) = -\sigma \beta V(t) I(t) - (\mu + \theta) V(t) \end{cases}$$
$$\begin{cases} S(nT^+) = (1 - \varphi) S(nT^-) \\ I(nT^+) = I(nT^-) & \text{if } t = nT, \\ V(nT^+) = V(nT^-) + \varphi S(nT^-) \end{cases}$$

where $S(t)$ is the number of susceptibles, $I(t)$ is the number of infected and $V(t)$ is the number of vaccinated in time t, the Greek letters are the parameters of the model.

In most epidemiological models, if the reproduction number $R_c < 1$ (number of secondary infected individuals caused by a single infected introduced into a wholly susceptible population), the solutions converges to a disease-free equilibrium. If $R_c > 1$, then the solutions converges to an endemic equilibrium (i.e. $I > 0$). Hence, a forward bifurcation appears at $R_c = 1$. But a dangerous case can also happen that an endemic equilibrium appears even for $R_c < 1$, and the solution converges to either the disease-free equilibrium or the endemic equilibrium depending on the initial value. This means a so called backward bifurcation at $R_c = 1$. It is known, that this is the case at constant imperfect vaccination.

In our talk, we present our results on the consequences of imperfection of vaccines at pulse vaccination. In particular, we study whether backward bifurcation can arise in a pulse vaccination model. Due to the strong nonlinearity of the model, we show both theoret ical results, and dynamic experiments and simulations prepared in Wolfram *Mathematica.*